EQUATION OF A LINE

1. Standard form of the equation of a line

An equation of the type : $Ax + By + C = 0$ is called **standard form of the equation of a line.**
Properties:

1. If A = O, then the equation obtained is $By + C = 0$, or $y = -C/B$, which is a line parallel to the x– axis, at a distance $-C/B$ from the origin.
2. If B = O, the equation is reduced to $Ax + C = 0$, or $x = -C/A$ which is a line parallel to the y-axis, at a distance $-C/A$ from the origin.
3. If $B= 0$, $C = 0$, the line equation is reduced to the type $Ax=0$, or $x=0$. It corresponds to the y-axis, and it is its equation.
4. If A = 0, C = 0, the line equation is reduced to the type $By = 0$, or $y=0$. The line corresponds to the x-axis and it is its equation.

2. Explicit form of the line equation

The equation $y = kx + b$, where $k = tg\,a$, is called **equation of a line in its explicit form** (fig. 4).

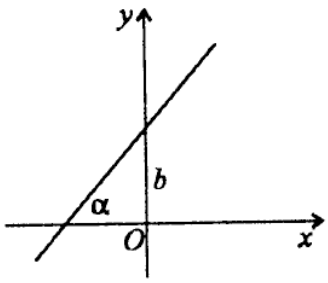

Fig. 4

The number k is called the slope of the line, and b-the intercept with the y-axis.

Example. Write the line equation for a line that goes through the origin of the coordinate system and is at an angle of 135° with the positive part of the x-axis.
Solution. n=0 , k=tg135°=-1 . So, y=-x is the line equation. It is the bisector of the II and IV quadrants.

3.Segment form of the line equation

The line equation in its segment form is:

$$\frac{x}{a}+\frac{y}{b}=1$$

where a and b are segment that the line cuts in the x and y axes respectively. (Fig. 5).

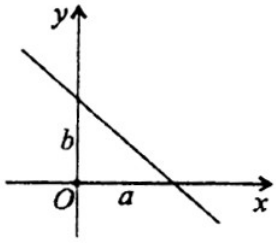

Fig.5

Example. Write the equation of a line if m and n are respectively $\frac{2}{3}$, $\frac{3}{4}$.

Solution.

$$\frac{x}{\frac{2}{3}}+\frac{y}{\frac{3}{4}}=1;\frac{3x}{2}+\frac{4y}{3}=1;9x+8y-6=0$$

4. Regular form of the line equation

The regular form of the line equation in its general form is

$$\frac{Ax+By+C}{\pm\sqrt{A^2+B^2}}=0$$

in which the sign before the square root is the opposite of the sign of the coefficient C.

5. Equation of a line passing through one and two points

The equation of a line passing through a single point M (x_1,y_1) having a slope k is:

$$y-y_1=k\,(x-x_1)$$

The equation of a line that passes through two points $M_1(x_1,y_1)$ and $M_2(x_2,y_2)$ has the following form:

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

Example. Writhe the equation of the line determined by points A(1,2) and B(5,4)

Solution.

$$y-2=\frac{4-2}{5-1}(x-1),\ y-2=\frac{2}{4}(x-1),\ y-2=\frac{1}{2}(x-1),2(y-2)=x-1,x-2y+3=0$$

6. Distance of a point to the line

The discance from the point $M(x_0,y_0)$ to the line $Ax + By + C = 0$ is calculated with the equation:

$$d=\left|\frac{Ax_0+By_0+C}{\pm\sqrt{A^2+B^2}}\right|$$

Example.Determine the distance from the point M(4,3) to the line $2x+y-2=0$

$$d=\frac{2\cdot4+1\cdot3-2}{\sqrt{4^2+3^2}}=\frac{9}{\sqrt{16+9}}=\frac{9}{\sqrt{25}}=\frac{9}{5}$$

7. Angle between two lines

The angle between the lines l_1, and l_2 (fig. 6) is determined with the equation:

$$tg\varphi=\frac{k_2-k_1}{1+k_1k_2}$$

where k_1, and k_2 are the slopes of the lines l_1 and l_2, respectively.

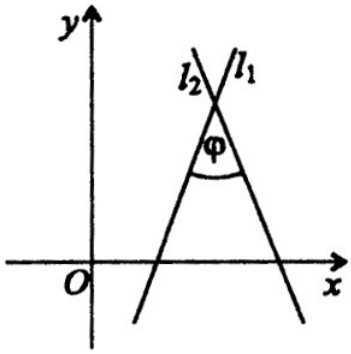

Fig. 6

From equation (1) the conditions for parallel or perpendicular lines are obtained. So , $l_1 || l_2$,

if $k_2 = k_1$, $l_1 \perp l_2$ if $1 + k_1 k_2 = 0$

Example. We have the lines $p_1 : 3x - 4y + 8 = 0, p_2 : 5y - 4mx - 7 = 0.$ Determine $m - ?$ such as the lines are parallel.

Solution. We get the equations in explicit form.

$$4y = 3x + 8, y = \frac{3}{4}x + 2, k_1 = \frac{3}{4}$$

$$5y = 4mx + 7, y = \frac{4m}{5}x + \frac{7}{5}, k_2 = \frac{4m}{5}$$

. Thus,

$$\frac{3}{4} = \frac{4m}{5}, 3 \cdot 5 = 4 \cdot 4m, 15 = 16m, m = \frac{15}{16}.$$